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ABSTRACT

A probabilistic approach is introduced to classify and diagnose erroneous rules of operation resulting from a variety of misconceptions ("bugs") in a procedural domain of arithmetic. The model is contrasted with the deterministic approach which has commonly been used in the field of artificial intelligence, and the advantage of treating the variability of errors in response is shown. Item response theory (IRT) is a useful model in integrating the theory of cognitive processes with educational practice. Erroneous rules of operation in signed-number subtraction problems can be represented as points in a geometric space by utilizing IRT (rule space). This approach seems promising in assessing the state of knowledge as reflected by erroneous rules and in utilizing the information obtained from behaviors of "bugs." (Author/PN)

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Computer-based Education Research Laboratory

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A PROBABILISTIC MODEL FOR DIAGNOSING MISCONCEPTIONS BY A PATTERN CLASSIFICATION APPROACH

KIKUMI K. TATSUOKA

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used in the field of artificial intelligence and shows an advantage in treating the variability of errors in responses. Item response theory (IRT) turned out to be a useful model in integrating the theory of cognitive processes with educational practice. In this paper, erroneous rules of operation in signed-number subtraction problems are represented as points in a geometric space by utilizing IRT. We named this space "rule space." This approach seems promising in assessing the state of knowledge as reflected by erroneous rules and in utilizing the information obtained from behaviors of bugs into educational evaluation.

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This study introduces a model, utilizing item response theory, for dealing with various rules that students use in solving problems. Siegler (1976, 1978) developed a rule assessment method for handling choice data -- he used this method in the context of cognitive development in the balance scale. Anderson (1974, 1981) developed a functional measurement methodology to research the assessment of algebraic integration rules. Wilkening and Anderson (1982) compared the two methods (Siegler's binary decision tree method and Anderson's functional method) and discussed their advantages and disadvantages. Wilkening and Anderson state that the binary decision tree methodology does not resolve the underlying problem of lack of an error theory to handle response variability. The functional measurement method allows for unreliability or variability in the responses and allows analysis of variance to assess a goodness of fit measure between rules and data. It seems, however, that both the methods are more suitable for investigating a basic foundation of knowledge structure and development rather than for conducting evaluative studies on performance data.

This study will introduce a measurement model using item response theory (IRT) for dealing with the misconceptions committed by many students. Although the purpose of the model is neither to discover an unknown source of misconception from responses nor to represent knowledge structure like the binary decision tree method, it has the capability of diagnosing many erroneous rules. The primary purpose of the model is to establish an interface between cognitive processes and psychometrics.

It is useful to know the transitional behavior of error types which may be due to a change of instructional methods, advancement of learning stages, or the stability and persistence of particular misconceptions. Such knowledge can help to evaluate instruction, measure the outcome of learning and obtain diagnostic information for designing remedial instruction which should be particular to the type of misconception. The model should be able to express various aspects of misconceptions quantitatively so that they can be statistically related to other measures like motivation or creativity.

First the model, which is named "rule space," will be introduced. The rule space is formulated by using IRT models so as to facilitate probabilistic treatments for "behaviors" of misconceptions. An index measuring "usualness" of responses will also be briefly described because it is used as one of the coordinates of rule space. Secondly, rule space will be illustrated with signed-number arithmetic data and the responses generated from various erroneous rules will be shown as points in the rule space. Then we will discuss a technique for assessing rules used inconsistently by a student due to "slips" or the instability of his/her misconceptions. Using pattern classification techniques (Fukunaga, 1972) to determine the student's latent state of knowledge (Tatsuoka & Tatsuoka, 1981) or to find his/her misconception(s) seems very useful when taking the variability of errors into account.

Rule Space

All erroneous rules of operation in signed-number arithmetic (that have already been discovered) can be expressed as points in a geometric space called "rule space." In other words, rule space is a geometric representation of the rules used by students. Before the formulation of the rule space is presented, the extended caution index (which measures the degree of anomaly in response patterns) will be briefly introduced.

Extended Caution Index (ECI)

A group of extended caution indices, which provides information from patterns of responses to test items not contained in the total score, was introduced by Tatsuoaka and Linn (1981, 1983). Similar indices based on IRT (Wright & Stone, 1977; Levine & Rubin, 1979) were introduced as identifiers of "guessing, sleeping, fumbling and plodding" (Wright & Stone, 1977, p. 110) or "so atypical...that his or her aptitude test score fails to be a completely appropriate measure" (Levine & Rubin, 1979, p. 269). Statistical properties of the ECIs have been investigated by Tatsuoaka and Tatsuoaka (1982). The raw ECIs are standardized (SECI) by subtracting their conditional expectations then dividing them by their conditional standard error. By so doing, SECI provide values comparable at two different levels of person parameters.

The values of the ECIs are calculated by first constructing two matrices; one is a binary score matrix (y_{ij}) , $i=1, \dots, N, j=1, \dots, n$ where N is the number of students and n is the number of items in a test. The other is a probability matrix with elements P_{ij} , which values of a logistic function with one, two or three parameters, defined as

$$P_{ij} = c_j + \frac{1-c_j}{1+\exp[-Da_j(\theta_i-b_j)]}$$

where c_j is the guessing parameter, a_j is the item discriminating power, b_j is the item difficulty, and θ_i is person i 's ability or achievement level (Lord & Novick, 1968; Lord, 1980).

In practice, the estimated P_{ij} obtained by substituting a_j , b_j , c_j and θ_i by their estimated item and person parameters in the logistic function can be used. One of the ECIs, ECI4, is defined as an index reflecting anomaly of an actual response pattern at a given level of ability θ_i . It is the complement of the ratio of two covariances: the numerator is the covariance of the i th row vector, y_i , of (y_{ij}) and the i th row, P_i , of the probability matrix (P_{ij}) ; the denominator is the covariance of the column-mean vector of, $G = (G_1, G_2, \dots, G_n)$, and the i th row vector P_i , both of (P_{ij}) .

That is,

$$ECI4 = 1 - \frac{\text{cov}(y_i, P_i)}{\text{cov}(G, P_i)}$$

where

$$G_j = \frac{1}{N} \sum_{i=1}^N P_{ij}$$

The conditional expectation and variance of ECI4 are given by

$$E(ECI4|\theta_i) = 1 - \frac{\text{Var}(P_i)}{\text{cov}(G, P_i)},$$

$$\text{Var}(ECI4|\theta_i) = \frac{2a_j^2(P_{ij} - T_i)^2}{n^2 \text{cov}^2(G, P_i)}.$$

Thus, the standardized ECI4 is given by

$$ECI4_z = \frac{n \cdot \text{cov}(P_i - y_i, P_i)}{\sum_{j=1}^n \sigma_{ij}^2 (P_i - T_i)^2}$$

where $T_i = \frac{1}{n} \sum_{j=1}^n P_{ij}$, the raw-mean vector of (P_{ij}) and $\sigma_{ij}^2 = P_{ij}(1 - P_{ij})$, variance of item j at the level i .

Tatsuoka and Tatsuoka (1982) showed empirically that the standardized ECI4 (SECI4) has an appropriate normal distribution. This is not surprising because ECI4 is a weighted arithmetic mean of P_{ij} , $j=1, 2, \dots, n$, while the appropriateness measures developed by Levine and Rubin (1979) and Drasgow (1982), correspond to a geometric mean of P_{ij} . Both the extreme tails of the distribution correspond to more unusual response patterns while the points in the middle indicate the usual, typical response patterns. Harnisch and Tatsuoka (1983) examined empirically the relationship between SECIs and total scores, finding that SECI correlates nearly zero with the total scores, both linearly and curvilinearly.

Component scoring: decomposing the regular scoring procedure of "right" or "wrong" into finer components

Many erroneous rules in arithmetic can produce the right answer for a given item (Van Lehn, 1982; Birenbaum & Tatsuoka, 1983a; Tatsuoka & Tatsuoka, 1982, 1983; Davis 1980). For example, the item $-16 - (-4)$ can have the right answer by the following three erroneous rules: (1) always subtracting the two numbers and taking the sign of the number with the larger absolute value; (2) changing the minus operation sign to addition, misunderstanding the parentheses as the bars of absolute value and then applying the right rule for addition; (3) converting the subtraction to an addition problem by changing the sign of the second

number, then subtracting the smaller absolute value from the larger absolute value and taking the sign of the first number to the answer. These three erroneous rules, which are committed by a substantial number of seventh graders (Birenbaum & Tatsuoka, 1983), produce the right answer for all subtraction problems in which the first number has the larger absolute value. But if we give a second item $4 - (-16)$, then rule (1) produces -12 and rules (2) and (3) yield the answers of $+20$ and $+12$, respectively. Therefore, if we select an appropriate set of items, each rule would correspond to a unique set of responses to those items. It is not always true, however, that the traditional scoring of "right" or "wrong" for responses to the items produces a unique set of binary response patterns corresponding to each rule.

Tatsuoka and Baillie (1982) pointed out that there are several erroneous rules whose response patterns by the traditional scoring procedure are identical but which can be distinguished by decomposing the unit of the answer into finer components. Tatsuoka and Tatsuoka (1981) listed the response patterns of 45 erroneous rules in signed-number arithmetic also obtained from the regular scoring procedure. Some of the 45 binary patterns of 16 items are identical although the descriptions of the erroneous rules which produced these identical patterns are not. There is no way to distinguish two such different rules just by looking at their binary response patterns.

However, all the erroneous rules discovered so far in signed-number addition and subtraction problems can be expressed uniquely as sets of the binary response patterns resulting from the component scoring procedure obtained by decomposing the regular scoring procedure into

finer components -- e.g., the sign part of the answer for a given item and the absolute value part of the answer in signed-number problems, or, for a fraction problem, the three components (whole number, numerator and denominator) of the answer. The regular response patterns are elementwise products of the component response patterns. Table 1 describes this procedure with four examples of signed-number subtraction.

Insert Table 1 about here

Hereafter we will use this new scoring method, the component scoring procedure, in this study. Even though the rationale of component scoring is based on a signed-number study, it may be generalized to other domains of arithmetic or mathematics.

Rule space: True score and SECI4 for component response patterns

Remember that each student's regular response pattern obtained by regular scoring is decomposed into component response patterns so that his/her responses to the test items are now represented by component response patterns. The Euclidean space determined by the four variables, T_1^a , T_1^s , $SECI4_1^a$ and $SECI4_1^s$ will be called "rule space" hereafter. For example, the four rules of Table 1 are expressed as four points in the rule space.

Lord and Novick (1968) and Lord (1980) defined test characteristic function (or test response curve) as the average of n item response curves (or item characteristic functions) and denoted by $T(\theta)$.

That is,

$$T(\theta) = 1/n \sum_{j=1}^n P_j(\theta)$$

Table 1

The Binary Response Patterns of Three Different Scorings Generated by Four Rules

Items	Rule 16		Rule 32		Rule 12		Rule 46	
	Responses	R*S*A*	Responses	R S A	Responses	R S A	Responses	R S A
-3 - (-7) = +4	-4	0 0 1	+4	1 1 1	+4	1 1 1	+4	1 1 1
-2 - 8 = -10	+6	0 0 0	+6	0 0 0	+6	0 0 0	-10	1 1 1
5 - (-12) = +17	-7	0 0 0	+7	0 1 0	+17	1 1 1	-7	0 0 0
-11 - +8 = -19	-3	0 1 0	+3	0 0 0	-19	1 1 1	-19	1 1 1
9 - 4 = +5	+5	1 1 1	+5	1 1 1	+5	1 1 1	+5	1 1 1
-15 - (-9) = -6	-6	1 1 1	+6	0 0 1	-6	1 1 1	-6	1 1 1
-13 - 5 = -18	-8	0 1 0	+8	0 0 0	-8	0 1 0	-18	1 1 1
8 - (-16) = +14	+2	0 1 0	+2	0 1 0	+14	1 1 1	+2	0 1 0
-5 - +11 = -16	+6	0 0 0	+6	0 0 0	-16	1 1 1	-16	1 1 1
1 - 10 = -9	+9	0 0 1	+9	0 0 1	-9	1 1 1	-9	1 1 1

Rule 16: The student subtracts the smaller absolute values from the larger absolute value and takes the sign of the number with the larger absolute value in his/her answer.

Rule 32: The two numbers are always subtracted as seen in Rule 16 but the + sign is always taken in the answers.

Rule 12: The student converts -2 - 8 and -13 - 5 into -2 + 8 and -13 + 5, respectively, but the other eight items are converted to addition problems correctly. Then the right addition rule is used to answer them.

Rule 46: The student has a strange idea about the parentheses. Converts operation sign, -, to +, first. Then he/she follows the rule: if the signs of the two numbers are minus, then change the sign of the second number to a +; if the signs of the two numbers are not alike, then the sign of the second number becomes a minus.

*R Regular Scores

*S Sign-Component Scores

*A Absolute-Value-Component-Scores

Suppose a_{1j} , b_{1j} , $j=1,2,\dots,n$ be item parameters of the logistic model estimated by the maximum likelihood procedure obtained from absolute value component patterns and a_{2j} , b_{2j} , $j=1,2,\dots,n$ be item parameters estimated from sign components patterns. In other words, two binary data matrices, (y_{1j}^a) and y_{1j}^s which are obtained by component scoring procedures, are used to estimate the item parameter and for calculating $SECI4$ and T_1 (or $T(\theta_1)$) of the two components for each subject i . Thus, each subject's two component response patterns correspond to two ordered pairs: $(T_1^a, SECI4_1^a)$ for the absolute value and $(T_1^s, SECI4_1^s)$ for sign. Table 2 provides these ordered pairs for the four rules given in Table 1.

Insert Table 2 about here

An illustration of rule space with signed-number subtraction problem data.

A 40-item free-response test that comprises four parallel subtests of 10 items each in signed-number subtraction problems was administered to 172 eighth graders at a local junior high school (referred to as "Test 6" hereafter; more tests to be introduced later). The traditional scoring of right or wrong answers was decomposed into a two-component scoring procedure for the absolute-value and sign parts of the responses. Thus, the signs of the responses to the 40 items were scored right or wrong and so were the absolute values. The two component-response patterns are subjected separately to the estimation of item and person parameters of the two-parameter logistic model. The item parameters estimated by the maximum likelihood procedure are listed in Appendices I and II. Twenty complete erroneous rules that are often observed (at least 3 different students used them) are selected for this

Table 2

The Values of $(T_1^a, SECI4_1^a)$ and $(T_1^s, SECI4_1^s)$, the Ordered Pairs of True Score and Standardized Extended Caution Index for the Four Rules given in Table 1

Rules	T_1^a	$SECI4_1^a$	T_1^s	$SECI4_1^s$
Rule 16	.2966	-3.1570	.4488	-3.2816
Rule 32	.2966	-3.1570	.4555	2.4860
Rule 12	.7522	-2.6485	.8791	.2196
Rule 46	.7644	.0833	.8227	1.6241

study (Tatsuoka & Tatsuoka, 1981) and their component values of the rule space are calculated and listed in Appendix III.

These values of T_1^a , T_1^s , $SECI_1^a$ and $SECI_1^s$ for all students in the dataset as well as those for the twenty erroneous rules (Tatsuoka & Tatsuoka, 1981) are mapped into the rule space. Figure 1 shows a subspace whose coordinates are T_1^a and T_1^s . As has been mentioned earlier, a rule space is defined as a geometric representation of the rules (including the right rule and inconsistent application of two or more rules) used by the students.

Insert Figure 1 about here

Twenty small circles (o) in Figure 1 represent twenty different erroneous rules while the plus signs (+) stand for the student's responses to the 40 items. If the student responds to the 40 items by applying one of the twenty erroneous rules consistently, then his/her point should coincide with the circle representing the rule. There are two such points "B" in Figure 1. Most points do not show overlap, but some real responses are located in the vicinity of a rule.

Tatsuoka and Baillie (1982) generated data which simulate responses resulting from inconsistent application of a rule. One or two out of the 40 items do not follow a given erroneous rule and thus the component response patterns do not completely match the patterns produced by the erroneous rule. Twenty sets of simulation data based on the twenty different rules were generated and plotted on the space spanned by both the component true scores in Figure 2. Rules 16, 32, 12 and 46 and their simulated responses which cluster around corresponding rules are not separated well from each other in Figure 2. As can be seen in

Sign True Score

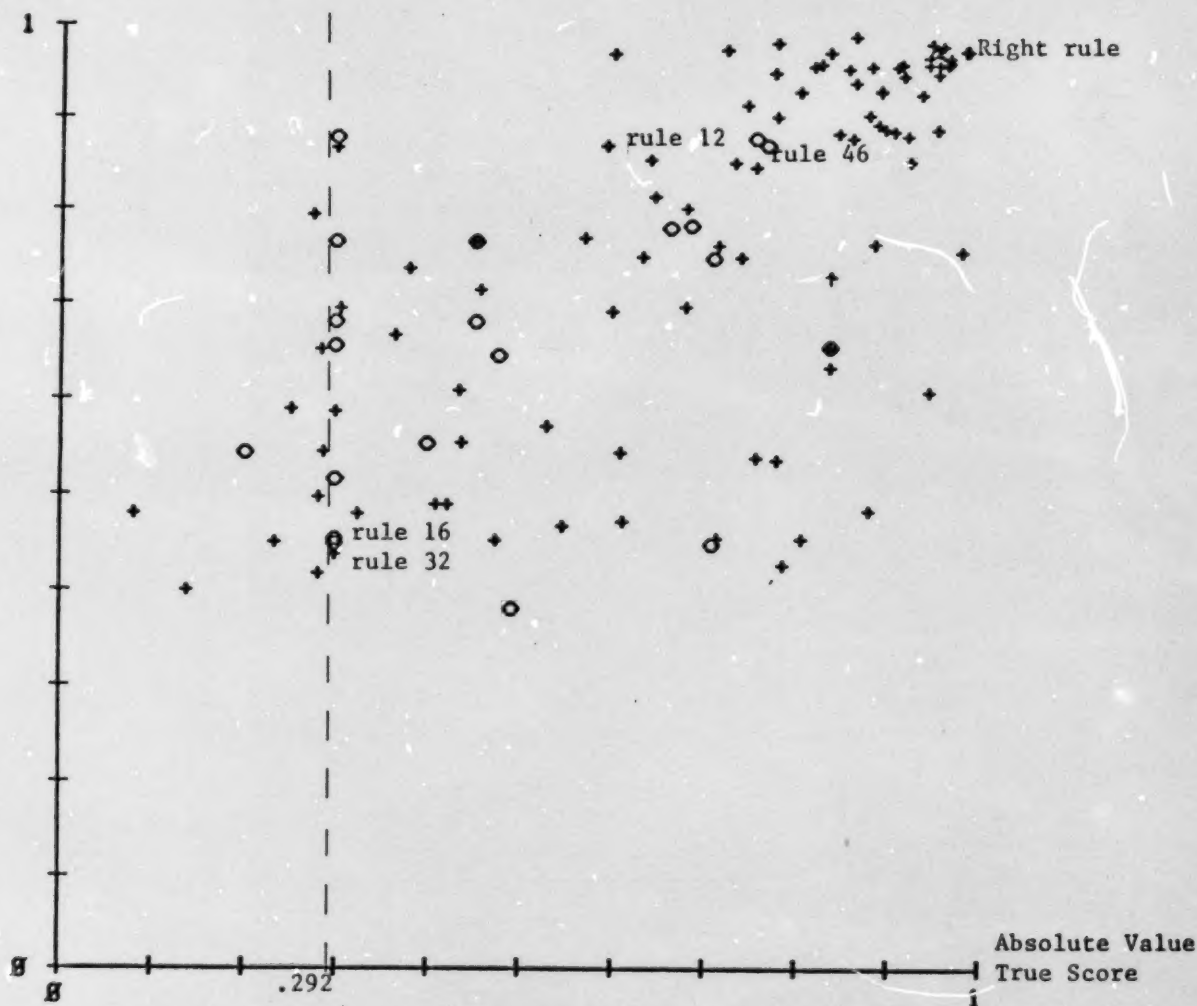


Figure 1: Plot of True Score for Absolute Value Component Against Sign Component, Real Data ("+") and Erroneous Rules ("o").

Figure 1, Rules 16 and 32, and 12 and 46 are already very close, respectively.

Insert Figure 2 about here

But, when plotted in terms of the sign true score against the standardized ECI4s obtained from the sign-component scores, four distinctly different clusters are formed in this space as shown in Figure 3a. In a similar figure, Figure 3b, the absolute value true score is plotted against the ECI4a, showing rules 12 and 46 distinctly separated.

Insert Figures 3a & 3b about here

It is apparent that the values of ECIs are capable of separating response patterns that have very close true scores or the same total scores.

Pattern Classification

In the previous section, Figure 3 showed the four erroneous rules (described in Table 1) and the non-consistent responses neighboring each of them forming four distinctly different clusters. By calculating a linear classification functions for each of the four clusters and setting the boundaries to divide the four regions, it is possible to classify the misconception underlying a new response by examining the region in which the new response falls -- with some probability of misclassification, of course. This is the traditional procedure for pattern classification and recognition problems to determine the category to which a new stimulus belongs (Fukunaga, 1972). Thus, we have transformed our problem, diagnosing an individual students' misconceptions, into a classification problem. Tatsuoka and Baillie have developed a computer program named SIGNBUG for diagnosing erroneous rules in signed-number arithmetic tests, but the logic of the algorithm

Sign True Score



rules 12 & 46



rules 16 & 32

14

Absolute Value
True Score

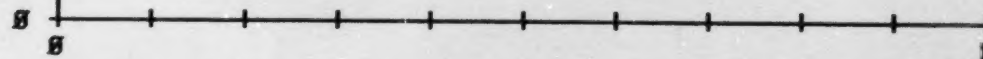
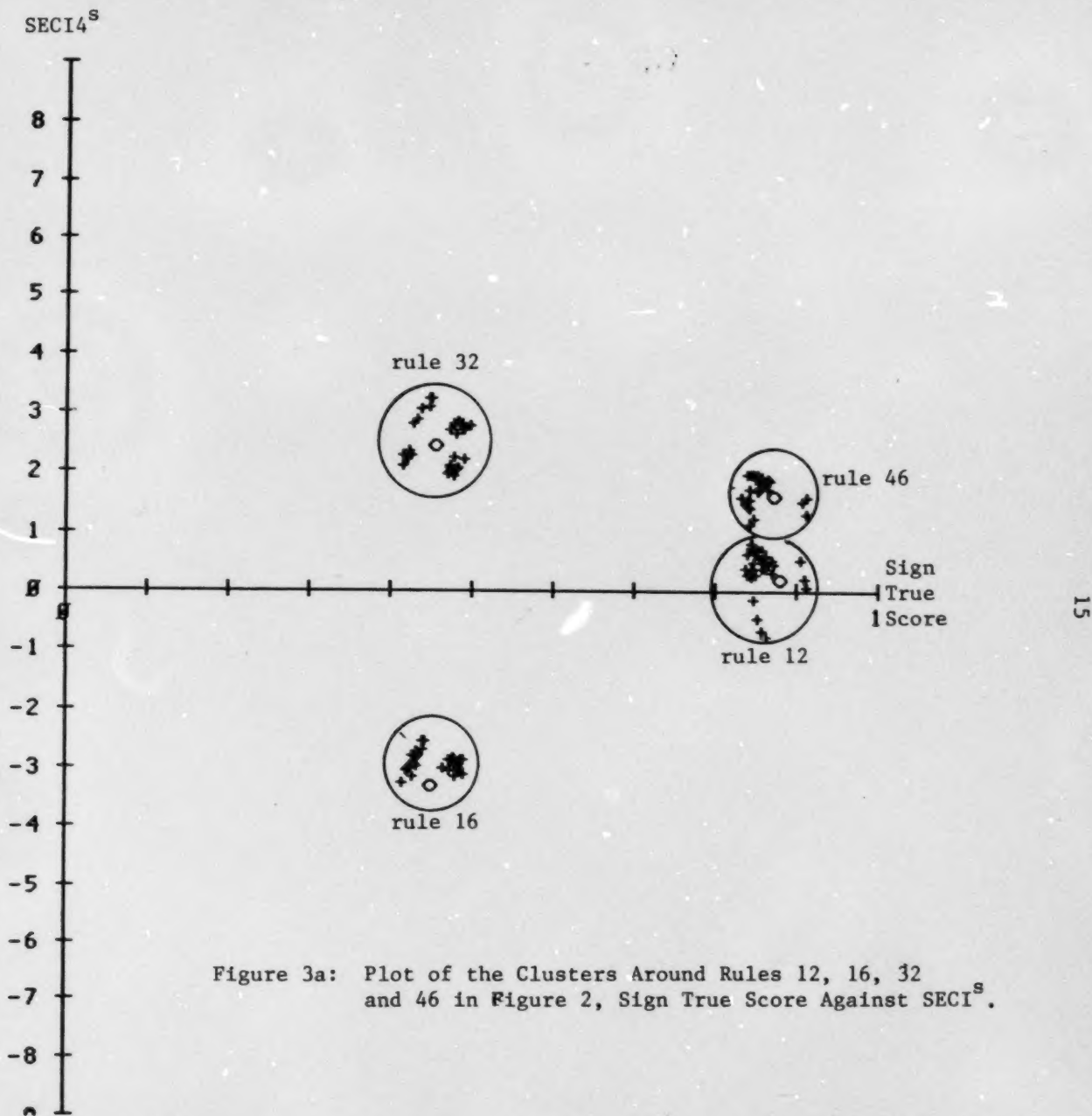


Figure 2: Plot of the Sign True Score Against Absolute Value for the Four Clusters Around Rules 12, 16, 32 and 46 in Figure 3.



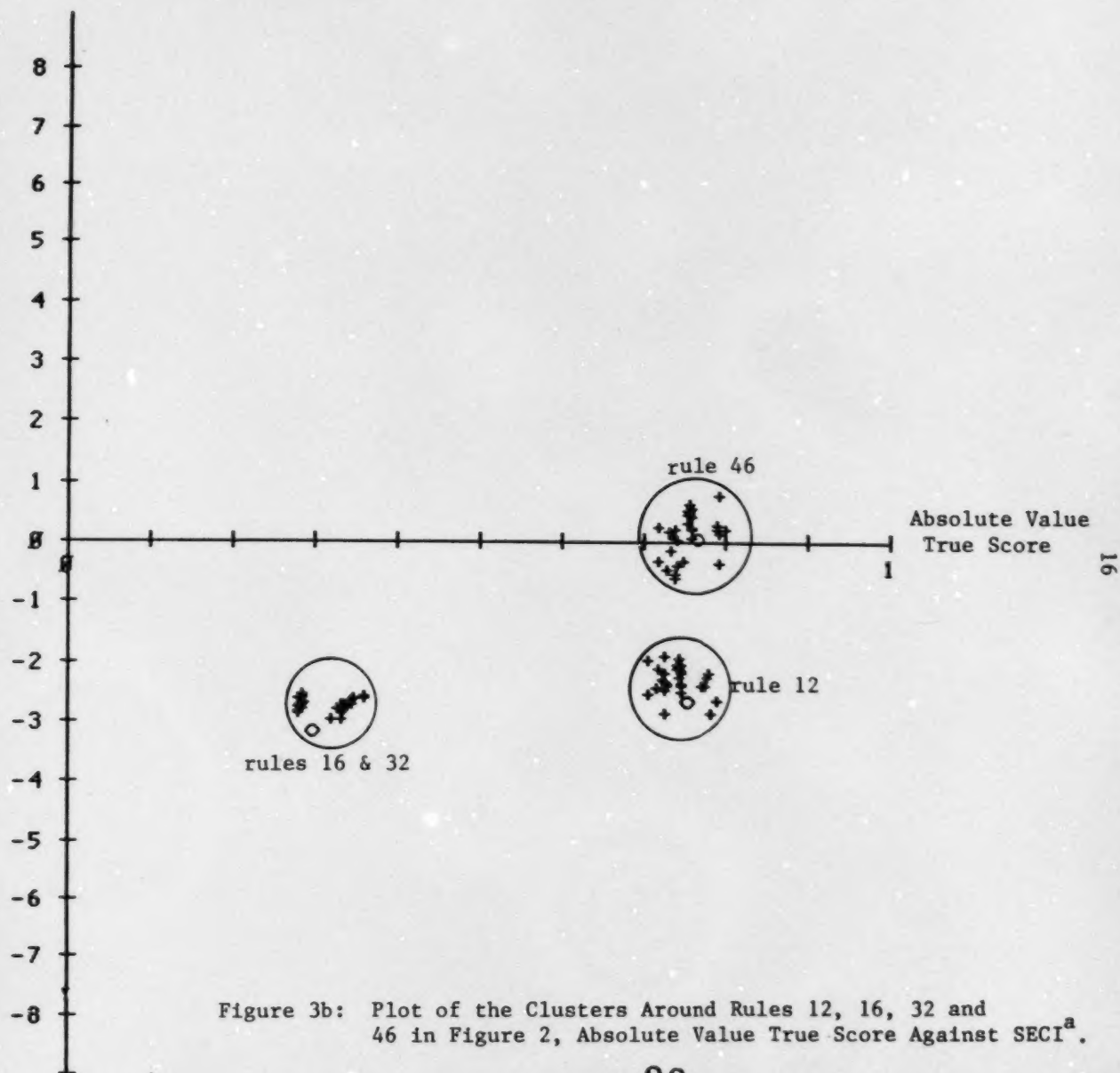


Figure 3b: Plot of the Clusters Around Rules 12, 16, 32 and 46 in Figure 2, Absolute Value True Score Against SECI^a.

is deterministic; therefore, if a student responds to an item without using a specific rule, then SIGNBUG cannot determine the rule.

As shown in Figures 2 and 3, the component response patterns yielded by using an erroneous rule consistently for the test items and the responses resulting from random "slips" of one or two items form a cluster; this is a nice feature of the rule space. An error theory that can handle response variability becomes applicable to our model. Since all erroneous rules that have been discovered so far in signed number arithmetic are represented by their unique component response patterns of absolute value and sign, these rules correspond to different ordered pairs of $(T_k^a, SECI4_k^a)$ and of $(T_k^s, SECI4_k^s)$, $k=1\dots K$. If each cluster of the erroneous rules could be separated from the rest of the clusters by a hyperplane in the four dimensional rule space of signed-number subtraction problems, then diagnosis of the responses resulting from random "slips" around a rule will be given by examining in which region (divided by the hyperplanes) the responses fall. This approach often is called "pattern classification." With the probabilistic approach of rule space and pattern classification it is possible to remedy the weakness of the deterministic approach taken in SIGNBUG without losing its strength.

In this paper, the classification boundaries of 20 clusters neighboring the 20 erroneous rules of signed-number subtraction problems are shown. The list of the 20 rules plotted in Figure 4 and their descriptions are given elsewhere (Tatsuoka & Tatsuoka, 1981), and Figure 5 shows the 20 clusters around the 20 rules. A stepwise discriminant analysis (BMDP7M) was used to determine the classification functions and Table 3 summarizes the results.

Insert Figures 4 & 5 about here

Insert Table 3 about here

Nineteen of the rules are perfectly classified, without any error of classification, and only rule (45) has one out of the 31 samples misclassified. Four independent variables -- absolute value, and sign true scores, and SECI4 for absolute value and signs -- were used in the analysis.

Data Analysis

Changes of responses over time for individual students. The 40-item open-ended test for subtraction problems of signed number arithmetic was administered four times to the students in a local junior high school in 1981. The first test was administered before instruction on the subject was given to the eighth graders and is referred to as "Test 3." The instruction (lessons) were written on the computer-based education system (PLATO^R) at the University of Illinois. The lessons are each almost one hour long. Two different instructional methods -- one based on the number line and the other, which relies heavily on verbal ability, using the postman stories (Davis, 1964) -- are given to two randomly selected groups. We will refer to the number line group as Group 1 and the postman group as Group 2 hereafter.

The second test (Test 4) was administered after the students completed the two PLATO lessons. Subsequently, a regular class teaching subtraction skills was held. The teachers adopted a method using verbal rules (described in Birenbaum & Tatsuoka, 1980; Tatsuoka, 1981) and drilled the students for two weeks. Although they referred to the number line method in a systematic way, they did not mention the postman stories at all. After two weeks of classroom instruction, the third test (Test 5) was

Sign True Score

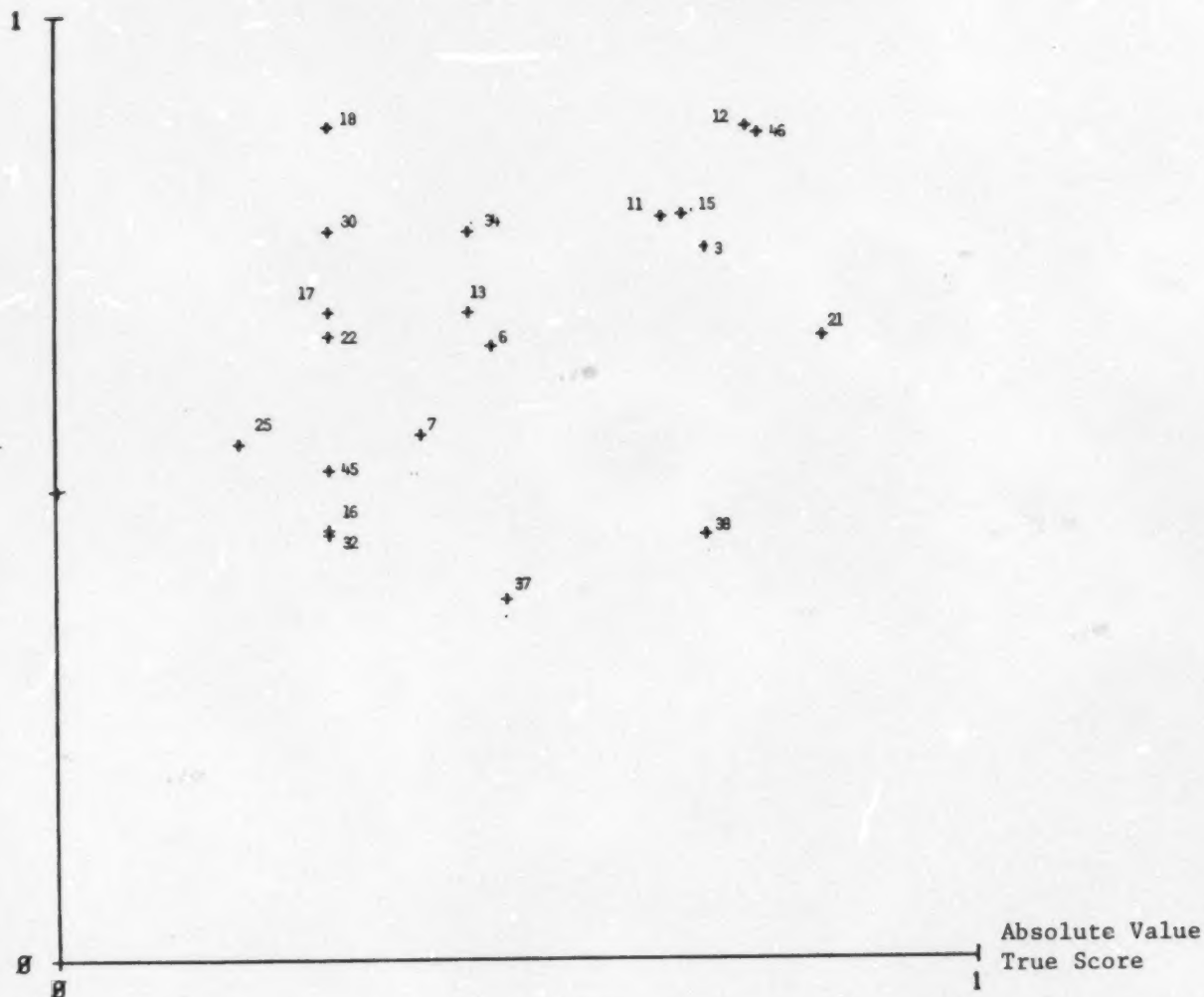


Figure 4: Twenty Erroneous Rules of Signed-Number Subtraction Problems

Sign True Score

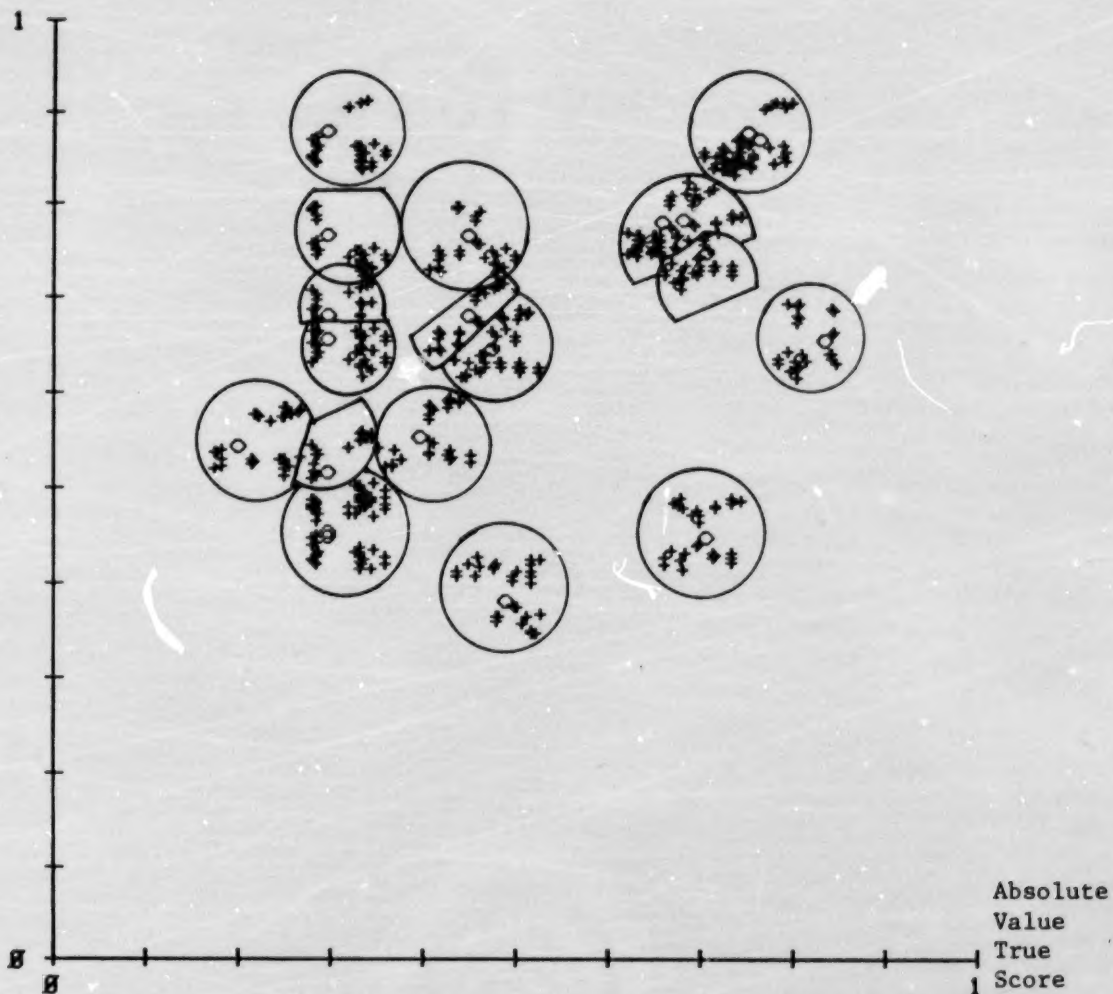


Figure 5: Twenty Clusters of the Responses Neighboring the Twenty Rules

Table 3

Classification Matrix: Number of Cases Classified into Group and Percent Correct

Rule	% Correct	Rule																			
		3	6	7	11	12	13	15	16	17	18	21	22	25	30	32	34	37	38	45	46
3	100	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	100	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	100	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	100	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	100	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	100	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	100	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0	0
16	100	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0	0
17	100	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0	0
18	100	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0	0
21	100	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0	0
22	100	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0	0
25	100	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0	0
30	100	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0
32	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0
34	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0
37	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0
38	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0
45	96.8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	30	0
46	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31
Total	99.8	31	31	31	31	31	31	31	32	31	31	31	31	31	31	31	31	31	31	30	31

administered. The fourth test (Test 6) was given after the students completed multiplication and division of signed numbers. Test 6 was mentioned earlier.

Since the 40-item test is composed of four parallel subtests of ten items, the comparison of errors committed by the student across the four subtests can be carried out by plotting the responses to the ten items four times in the rule space obtained by the 10-item subtest. That is, the student's responses to the 40 items yield 4 points, each of which corresponds to one of the four parallel subtests. Table 4 shows the component values of the rules space for two students A and B. T_{A1} , T_{A2} , T_{A3} and T_{A4} are obtained by averaging estimated logistic probabilities $P_j(\theta_A)$ over 10 items in each subtest,

$$T_{AK} = \frac{1}{10} \sum_{j=1}^{10} P_j(\theta_A), K=1,2,$$

ECI^4_{A1-4} are calculated by using 10 items in each subtest. Thus, four sets of two ordered pairs are obtained from Student A's responses to the four parallel subtests. Note that the coordinates of the plots in Figure 1 are based on 40 items, but the coordinates of the points in Figure 6 are obtained from 10 items. Student A studied the number line method (Group 1) and student B studied the postman stories (Group 2). Their performances on the four subtests are shown in Table 4. Interpretation of the changes made by Students A and B across the four subtests, over the four different stages of learning designated by Tests 3 through 6 are summarized in Appendix IV. Their rule space representations are given in Figures 6 and 7.

Insert Table 4 about here

Insert Figures 6 & 7 about here

Table 4

The Performances by Students A and B on the Four Subtests of Tests 3, 4, 5, and 6

Subtest 1					Subtest 2						
		Test 3		Test 4				Test 3		Test 4	
Item		A	B	A	B	Item		A	B	A	B
1.	-6 - (-8)	2	-2	14	2	17.	-1 - (-10)	-9	-11	-9	9
2.	-7 - 9	2	-2	2	2	18.	-2 - 11	9	-13	9	-13
4.	1 - (-10)	-9	-9	-9	11	20.	3 - (12)	-9	-15	-9	15
6.	-12 - +3	-9	-9	-9	9	22.	-13 - +4	-9	-9	-17	9
7.	8 - 6	2	2	2	2	23.	7 - 5	2	2	2	2
8.	-16 - (-7)	-9	-9	-9	-9	24.	-12 - (-10)	-2	-2	2	2
9.	-12 - 3	-9	-9	-9	-15	25.	-6 - 4	-2	-2	-2	-10
12.	9 - (-7)	2	-2	2	16	28.	6 - (-4)	2	-2	2	10
13.	-3 - +12	9	-15	13	-15	29.	-2 - +11	9	9	9	-13
16	2 - 11	9	-10	9	-9	32.	5 - 14	9	-10	9	-9

Subtest 3						Subtest 4					
		Test 3		Test 4				Test 3		Test 4	
Item		A	B	A	B	Item		A	B	A	B
33.	-3 - (-5)	-2	-8	2	2	49.	-2 - (-11)	-9	-13	-13	9
34.	-4 - 6	2	-2	2	-10	50.	-5 - 14	9	-19	9	-19
36.	5 - (-7)	-2	-12	-2	12	52.	6 - (-8)	-2	-3	-2	14
38.	-14 - +5	-9	-9	-19	-19	54.	-15 - +6	-9	-9	-9	-21
39.	4 - 2	2	2	2	2	55.	9 - 7	2	2	-2	2
40.	-11 - (-2)	-9	-9	-13	-9	56.	-7 - (-5)	-2	-2	-12	2
41.	-13 - 4	-9	-9	-17	-17	57.	-9 - 7	-2	-2	-2	-16
44.	10 - (-1)	9	9	11	11	60.	13 - (-4)	9	9	9	17
45.	-7 - +9	2	3	2	-16	61.	-4 - +6	2	3	2	-10
48.	7 - 16	9	-10	9	-9	64.	4 - 13	9	-10	9	-9

(cont.)

Table 4 (cont.)

Subtest 1					Subtest 2						
		Test 5		Test 6				Test 5		Test 6	
Item		A	B	A	B	Item		A	B	A	B
1.	-3 - (-7)	4	-10	4	4	17.	-2 - (-11)	9	-13	9	9
2.	-2 - 8	-10	6	-10	-10	18.	-3 - 15	-18	12	-18	-18
4.	5 - (-12)	17	-7	17	17	20.	6 - (-11)	17	-17	17	17
6.	-11 - +8	-19	-3	-19	-19	22.	-12 - +4	-16	-8	-16	-16
7.	9 - 4	5	3	5	5	23.	9 - 6	3	15	3	3
8.	-15 - (-9)	-6	-24	-6	-6	24.	-16 - (-10)	-6	-26	-6	-6
9.	-13 - 5	-18	-8	-18	-18	25.	-5 - 3	-8	-8	-8	-8
12.	8 - (-6)	14	2	-2	14	28.	6 - (-4)	10	2	10	10
13.	-5 - +11	-16	6	-16	-16	29.	-4 - +11	-15	-7	-15	-15
16.	1 - 10	-9	11	-9	-9	32.	6 - 12	-18	18	-6	-6

Subtest 3					Subtest 4						
		Test 5		Test 6				Test 5		Test 6	
	Item	A	B	A	B		Item	A	B	A	B
33.	-2 - (-8)	6	-10	6	6	49.	3 - (-12)	9	-15	9	9
34.	-3 - 9	-12	-12	-12	-12	50.	-6 - 13	-19	7	-7	-19
36.	4 - (-9)	13	-13	13	13	52.	4 - (-7)	11	-2	11	11
38.	-11 - +7	-18	-5	-18	-18	54.	-16 - +5	-21	-11	-21	-21
39.	8 - 5	3	13	3	3	55.	6 - 4	2	10	2	2
40.	-13 - (-4)	-9	-17	-9	-9	56.	-9 - (-5)	-4	-14	14	-4
41.	-15 - 3	-18	-18	-18	-18	57.	8 - 4	-12	-4	-12	-12
44.	11 - (-1)	12	10	12	12	60.	12 - (-5)	17	7	17	17
45.	- 6 - +8	-2	2	-14	-14	61.	-7 - +8	-15	1	-15	-15
48.	5 - 13	-8	18	-8	-8	64.	6 - 14	-8	20	-8	-8

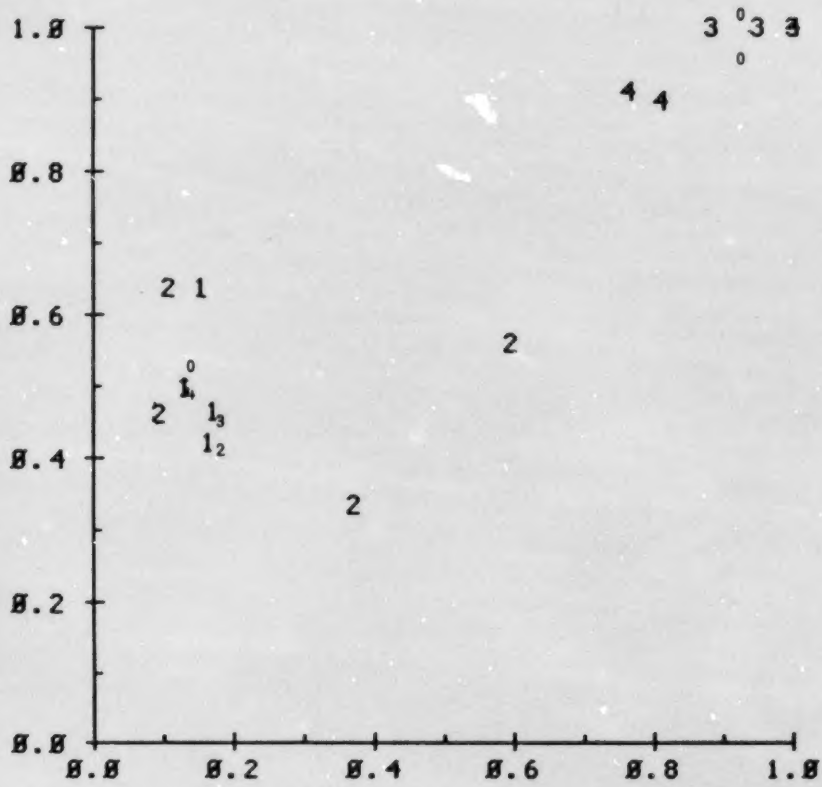


Figure 6: Plot of Student A's Performance Over Tests 3, 4, 5, and 6, Numbered 1, 2, 3, and 4, respectively

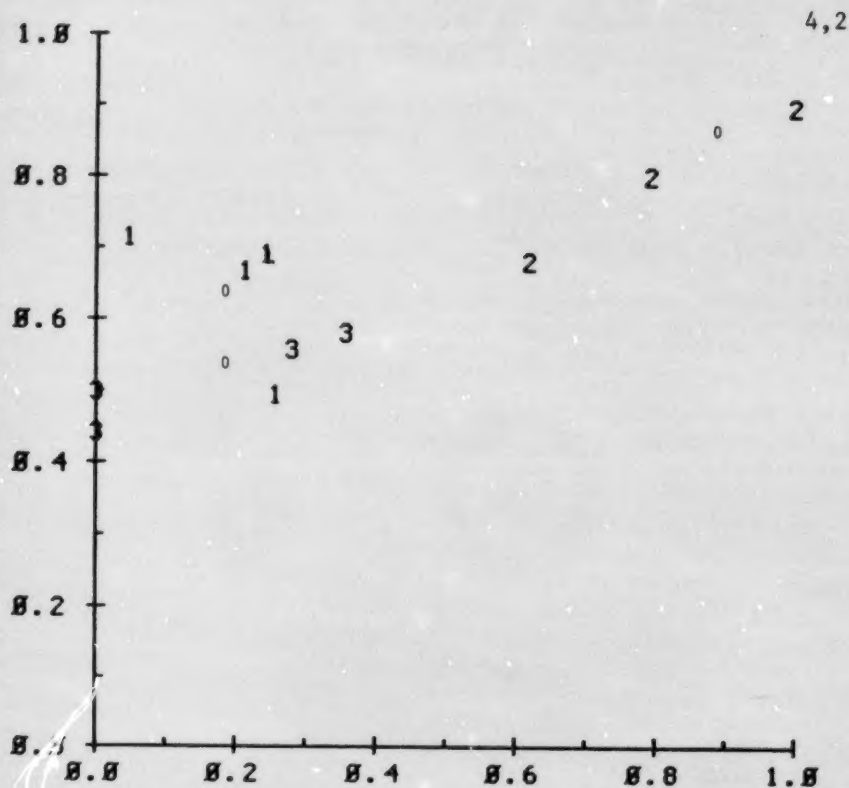


Figure 7: Plot of Student B's Performance Over Tests 3, 4, 5, and 6, Numbered 1, 2, 3, and 4, Respectively

The progress shown by student A over the four tests is normal because his average points obtained on the four subtests (marked by "o") in each of the four tests (Tests 3 through 6) gradually moved toward the top-right corner of Figure 6. The use of the right rule is designated by the point (1,1) at the top right corner. Variation of the four points in a test is due to the variability of responses as well to sampling errors. Since each wave of subtests consists of ten carefully chosen parallel items, if performance on the test is perfectly consistent over the four subsets, then the estimated parameter should be identical for 4 parallel items. But actual data used for estimating the item and person parameters by the maximum likelihood procedure of the two parameter logistic model was not so. Therefore, the four points of the four tests do not coincide perfectly as a single point. For example, student A produced identical responses for the first wave of subtests in Tests 3 and 4. Yet, the points that appeared in Figure 4 are slightly different. Also, the performances on the second, third and fourth subtests of the first test are identical, committing the same erroneous rules, but the points designating these responses are slightly different; l_2 , l_3 , and l_4 as shown in the figure.

Student B mastered the computational skills fairly well after he studied the PLATO lessons (postman stories). After the class, however, his performance was affected by the different teaching methods and he displayed confusion when converting subtraction operations into addition operation. The postman-stories approach does not teach the steps in converting subtraction to addition in a step-by-step fashion as the teacher's verbal rules do. Thus his errors (diagnosed by our computer

program SIGNBUG) clearly showed that he did carry out newly converted addition problems correctly (the verbal rule is first to change the operation sign of "-" to "+" and then to change the sign of the second number), but incorrectly converted subtraction into addition problems.

Changes of responses at different points in time

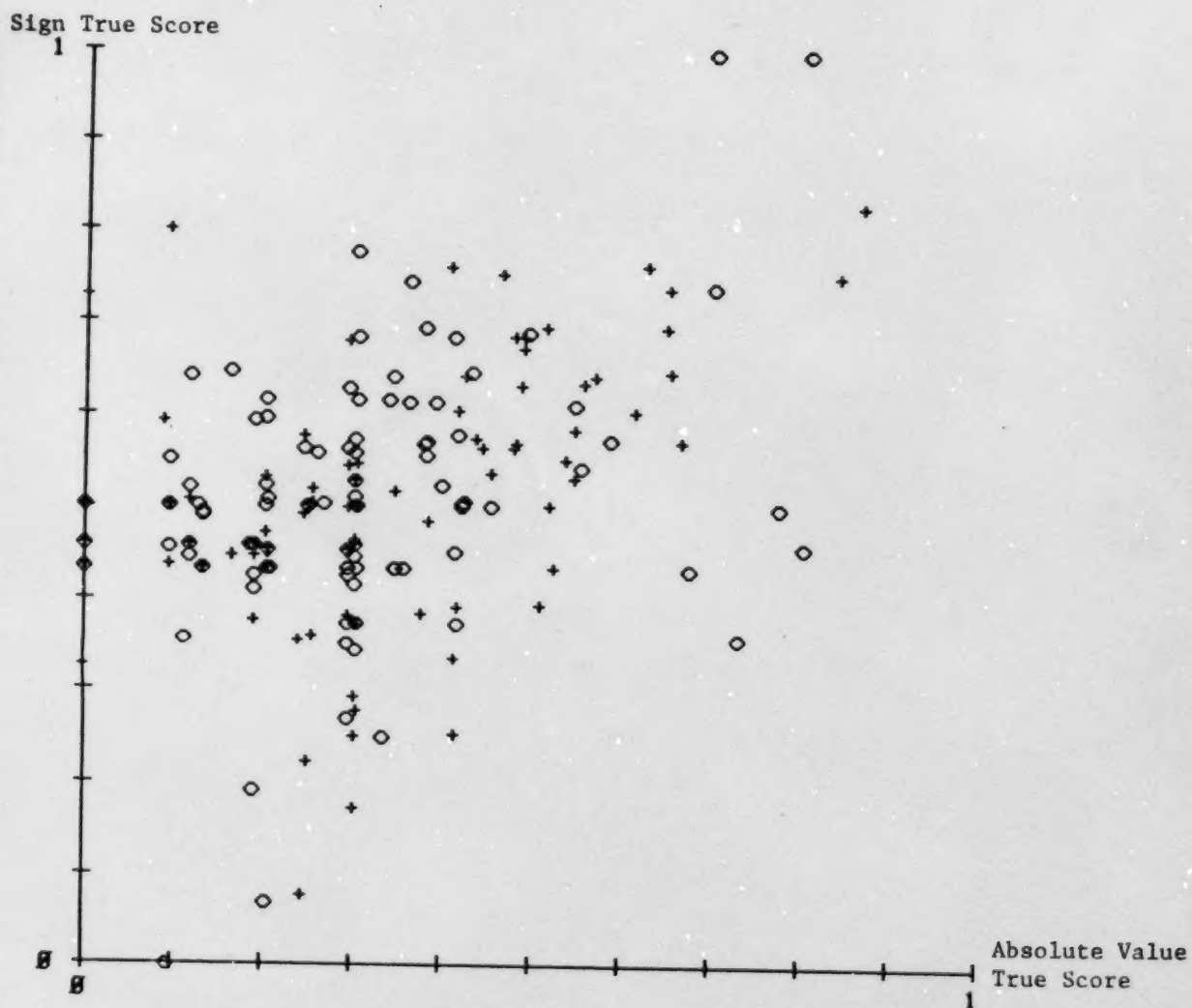
Figures 8 through 11 are plots of the responses made by all students who took Tests 4 through 6. The coordinates of the plots in the figures are the true scores (Lord and Novick, 1968) of the two components scores, absolute values and signs. The trend of changes in the points of the four tests is clear: As the stages of learning advance toward mastery of the right rule, the cluster moves toward the right top corner [(1,1) represents the use of the right rule] of the space spanned by two component true scores. The points from Test 5 in Figure 9 cluster most closely to the point (1,1), the right rule.

Insert Figures 8, 9, 10 & 11 about here

But the points from Test 6 in Figure 11 are no longer clustering as closely to the top right corner of the space as the points of Test 5 are. Learning new materials (i.e., multiplication and division of signed numbers) after the completion of the subtraction unit affected the performances on Test 6.

Summary and Discussion

This study introduced a probabilistic model utilizing item response theory for dealing with a variety of misconceptions. The model can be used for evaluating the transition behavior of error types, advancement of learning stages, or the stability and persistence of particular misconceptions. Moreover, it can be used for relating the



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Figure 8: Plotting of All Students Participating in the Experiments (Test 3, Postman Stories "o" and Number Line Groups "+").

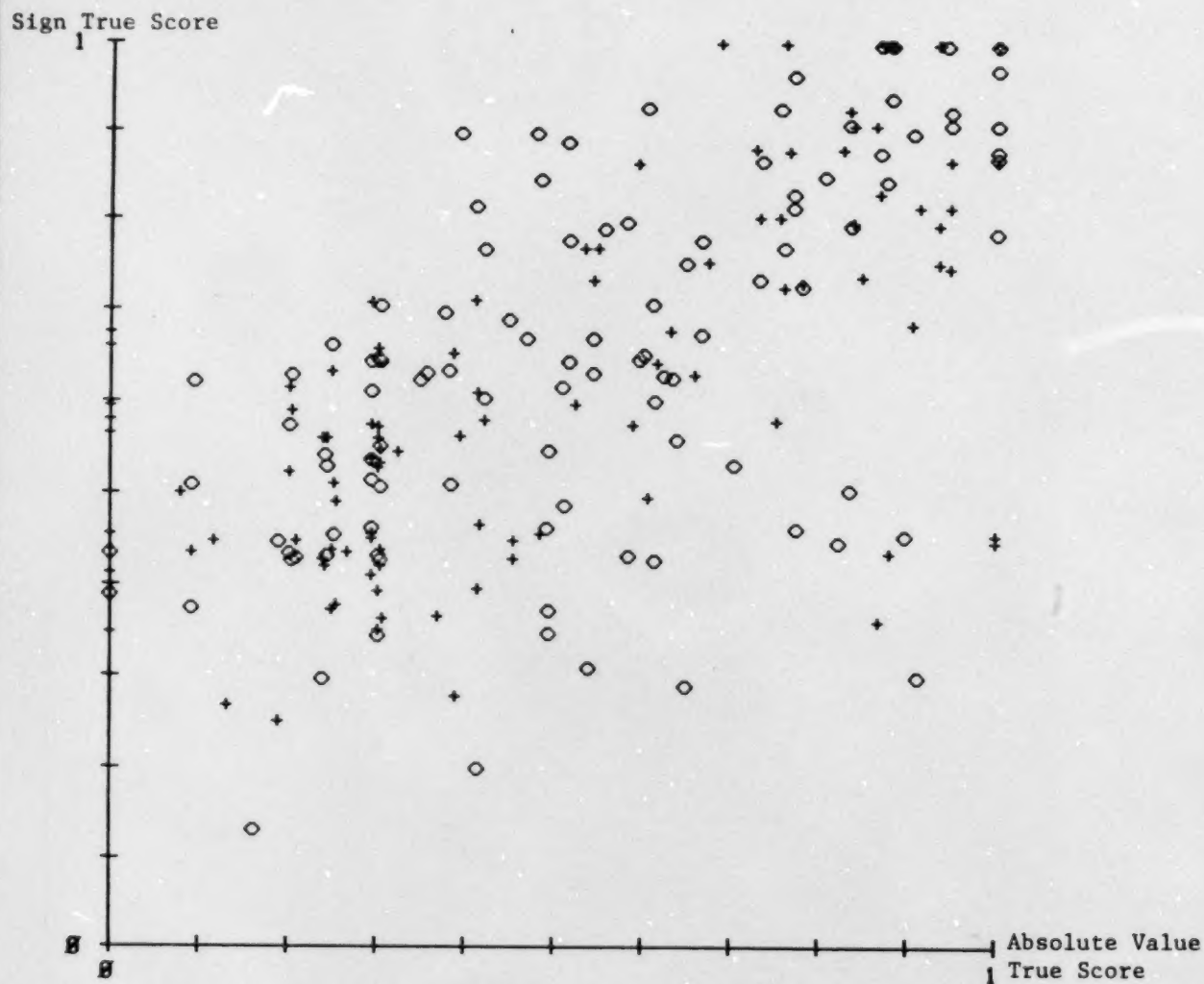
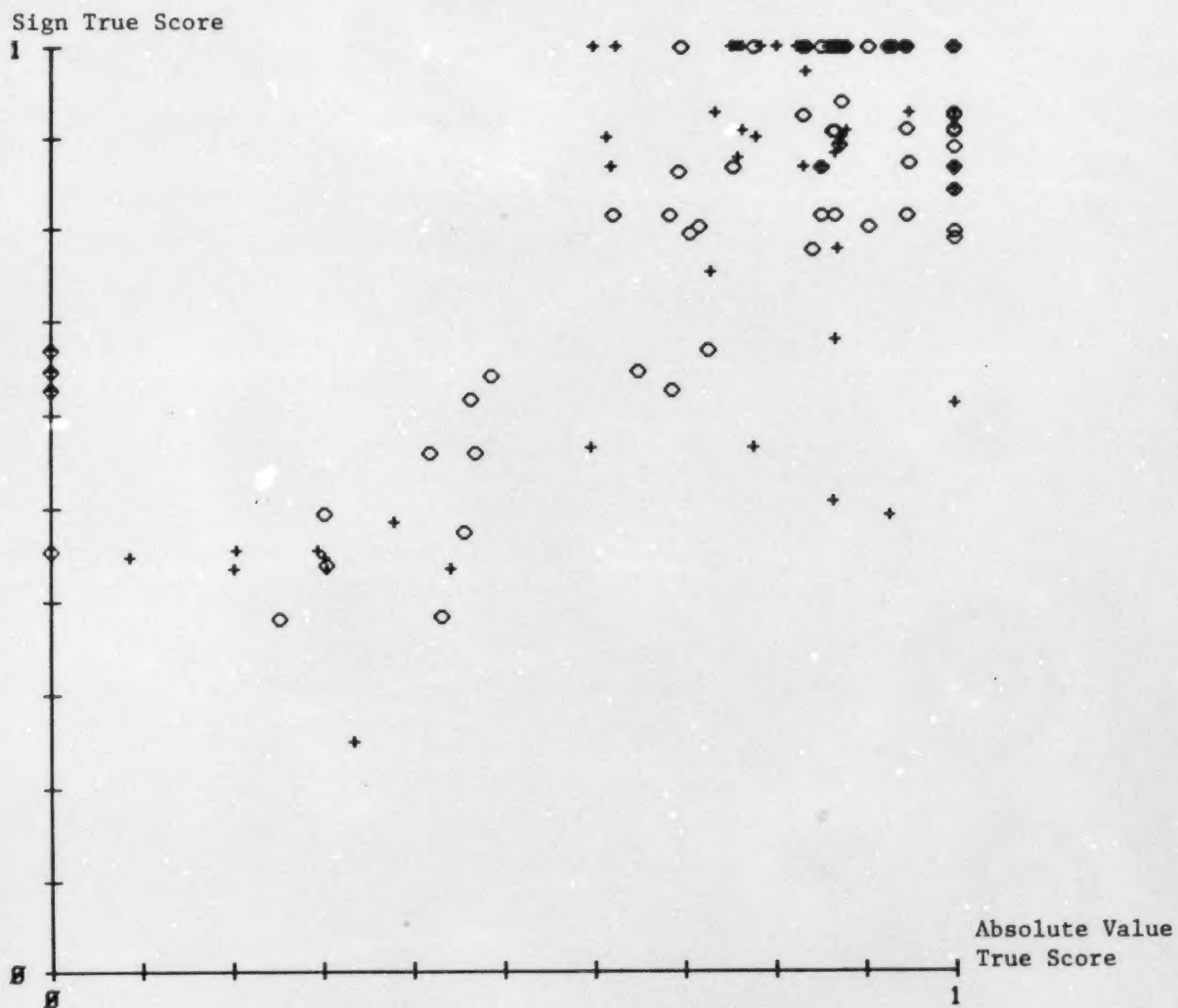


Figure 9: Plotting of All Students Participating in the Experiments
(Test 4, Postman Stories "o" and Number Line Groups "+").



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Figure 10: Plotting of All Students Participating in the Experiments (Test 5, Postman Stories "o" and Number Line Groups "+").

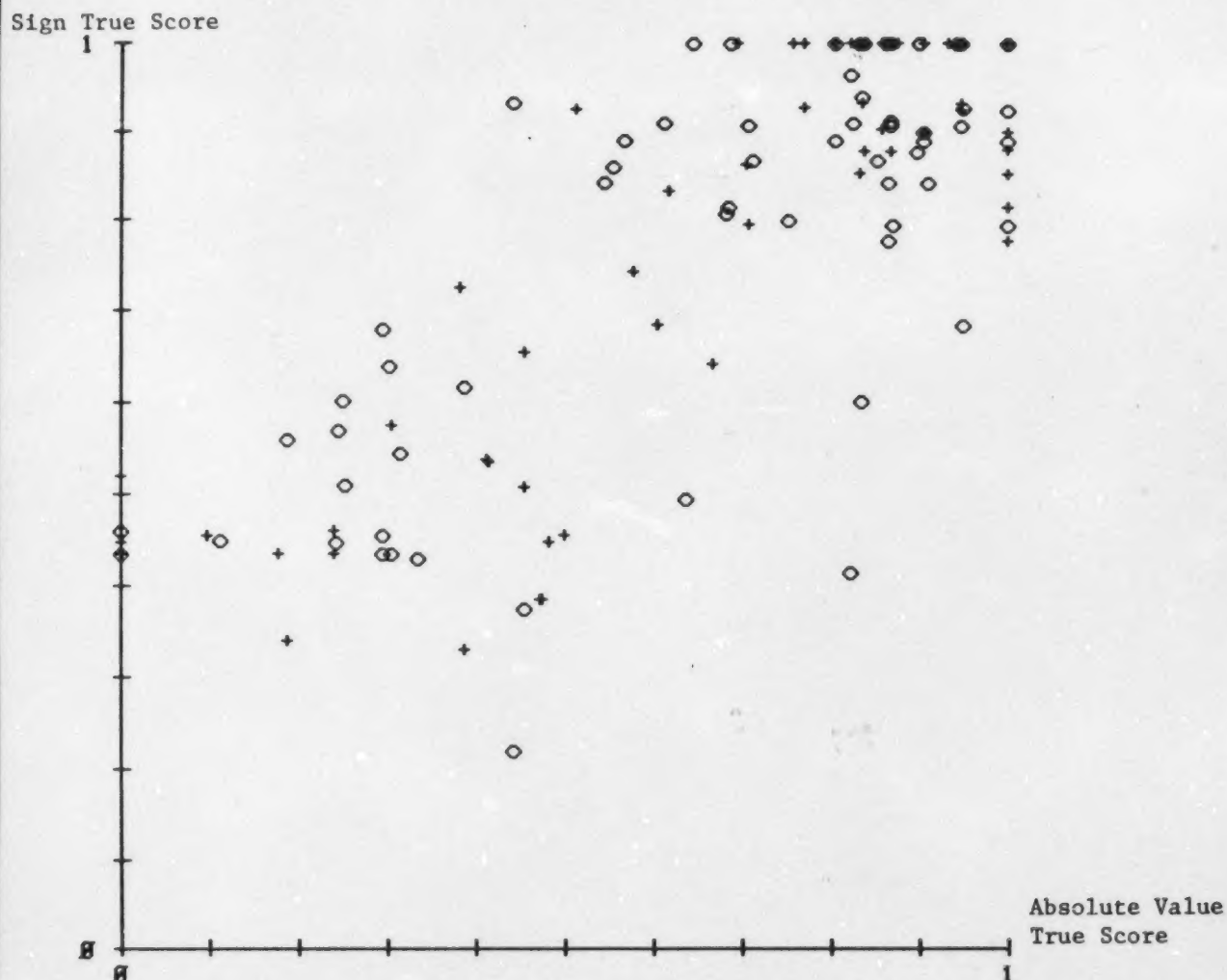


Figure 11: Plotting of All Students Participating in the Experiments
(Test 6, Postman Stories "o" and Number Line Groups "4").

"behaviors" of errors to other criterion measures such as creativity, anxiety and motivation.

One of several personal indices based on item response theory was used to formulate "rule space" which is a geometric representation of erroneous rules of operation. The index in question, ECI4, which is used primarily for detecting aberrant response patterns, has proved to be effective for separating clusters of response patterns from one another.

Each cluster comprises the response patterns yielded by some rule and its "slips" -- due to partially consistent application of that rule. The model enables us to apply pattern classification techniques to distinguish a cluster of response patterns around an erroneous rule from other clusters. Thus, the probability of misclassification should be obtainable. However, rigorous investigation along this line is left for subsequent investigation.

The examples in this study only suggest how the rule space approach works and the results of further statistical analyses are not discussed.

Appendix I

Values of Discriminating Index $\hat{a}$ s, Difficulties $\hat{b}$ s Obtained from Test 6,
Subtraction Problems: Absolute Value Scoring

item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$
1.	.277	-4.729	17.	.574	-1.925	33.	.592	-2.160	49.	.464	-2.280
2.	1.099	-.830	18.	.962	-.660	34.	1.416	-.439	50.	1.697	-.556
4.	1.409	-.579	20.	1.420	-.635	36.	1.481	-.454	52.	1.959	-.524
6.	.989	-.779	22.	2.524	-.429	38.	2.527	-.592	54.	1.620	-.656
7.	.574	-1.992	23.	.486	-2.352	39.	.619	-1.582	55.	.652	-1.812
8.			24.	.402	-2.172	40.	.454	-2.573	56.	.433	-2.336
9.	1.248	-.475	25.	2.031	-.413	41.	1.281	-.522	57.	1.449	-.407
12.	1.277	-.581	28.	1.476	-.917	44.	1.409	-.667	60.	1.471	-.684
13.	2.042	-.660	29.	1.934	-.642	45.	1.389	-.703	61.	1.514	-.738
16.	.609	-1.718	32.	.666	-1.664	48.	.446	-1.726	64.	.442	-1.932

Appendix II

Values of Discriminating Index $\hat{a}$ s, and Difficulties $\hat{b}$ s
Obtained from Test 6, Subtraction Problems: Sign Scoring

item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$	item	$\hat{a}$	$\hat{b}$
1.	1.172	-1.099	17.	1.280	-1.115	33.	1.202	-.974	49.	1.356	-.913
2.	.661	-1.251	18.	.987	-.867	34.	1.231	-.855	50.	1.255	-.851
4.	2.259	-1.080	20.	2.031	-1.052	36.	1.739	-.924	52.	1.762	-.786
6.	.646	-2.494	22.	.689	-2.387	38.	.871	-1.839	54.	.755	-1.993
7.			23.	.144	-9.898	39.	.277	-5.580	55.	.286	-4.288
8.	.812	-2.066	24.	.629	-1.993	40.	.684	-1.961	56.	.824	-1.516
9.	.667	-2.244	25.	.459	-2.843	41.	.557	-2.444	57.	1.075	-1.499
12.	.464	-2.302	28.	1.097	-1.651	44.	.899	-1.809	60.	.703	-2.355
13.	1.206	-.784	29.	1.400	-.836	45.	1.210	-.673	61.	1.560	-.827
16.	1.202	-.422	32.	1.064	-.544	48.	.877	-.503	64.	.662	.578

Appendix III

Coordinates of the 20 Erroneous Rules in the Rule Space for a 40-item Signed-Number Subtraction Test

Rule	Component	True Scores	SECI	Component* Response Patterns	θ
3	absolute	.7084	3.3315	0101101011	-.3550
	sign	.7510	1.3322	0101111111	-.5516
6	absolute	.4761	2.9877	0100101001	-.8889
	sign	.6486	-.1234	0101111101	-.8726
7	absolute	.3987	6.3432	0100001000	-1.1116
	sign	.5291	-2.3831	0101111100	-1.1457
11	absolute	.6837	.6337	1011010110	-.4124
	sign	.7843	-2.6399	1011111110	-.4322
12	absolute	.7522	-2.6485	1011110111	-.2459
	sign	.8791	.2196	1011111111	.0117
13	absolute	.4488	1.5525	1010010100	-.9617
	sign	.6818	-2.7285	1011111100	-.7733
15	absolute	.6591	3.0325	1110011100	-.4681
	sign	.7836	-2.4379	1111111100	-.4349
16	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.4488	-3.2816	0001111100	-1.4754
17	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.6818	-2.7285	1011111100	-.7733
18	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.8791	.2196	1011111111	.0117
21	absolute	.8338	3.4064	0111001110	.0051
	sign	.6584	3.9372	0101011011	-.8436
22	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.6584	3.9372	0101011011	-.8436
25	absolute	.2006	.8670	0000100001	-2.2077
	sign	.5457	-1.4002	0001111101	-1.1738
30	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.7664	-1.5839	0111111110	-.4978
32	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.4555	2.4860	1010100100	-1.4534
34	absolute	.4488	1.5525	1010010100	-.9617
	sign	.7664	-1.5839	0111111110	-.4978
37	absolute	.4891	1.5936	1001010010	-.8558
	sign	.3837	.8349	0100111000	-1.7053
38	absolute	.7084	3.3315	0101101011	-.3550
	sign	.4488	-3.2816	0001111100	-1.4754
45	absolute	.2966	-3.1570	1000110001	-1.5288
	sign	.5186	-1.7692	1001101100	-1.2549
46	absolute	.7644	.0833	1101111011	-.2133
	sign	.8727	1.6241	1101111111	-.0265

*Since the test consists of four parallel tests (i.e., each task has four parallel items), the response patterns of the first ten items are given here.

Appendix IV

Stories of Student A and B's performances on
the Four Different Learning StagesStudent A

Pretest (Test 3): He studied the number line method written the PLATO system for about an hour in January, 1980 when he was in the seventh grade. He had not been exposed to any kind of instruction related to signed numbers before. In September of 1980, he took a 64-item signed number test along with 40 other eighth graders before a revised version of the number line lesson on the PLATO system was given. His diagnosed rules are as follows:

- Subtest 1 -- His rule for taking an absolute value in the answer was to always subtract the smaller number from the larger number. His rule for taking a sign in the answer was to take the sign of the larger number. However, his application of the rule is only consistent to items 2, 4, 6, 7, 8, 9, 12, 13 and 16, as shown in Table 2.
- Subtest 2 -- His application of the rule described above became consistent to all the items in a test.
- Subtest 3 -- His rule was the same as that used in Subtest 2
- Subtest 4 -- His rule was the same as that used in Subtest 2

The test after the PLATO lesson (1 hour) was given (Test 4):

Test 4 was administered to Student A after he studied a number line lesson on the PLATO system.

- Subtest 1 -- He still subtracted the smaller number from the larger number and took the sign of the larger number for the items described in Subtest 1 of the pretest (items 2, 4, 6, 7, 8, 9, 12, 13 and 16).
- Subtest 2 -- He used basically the same rule but applied it to different subsets of items for both the sign and the absolute value operations.
- Subtest 3 -- He suddenly changed his rule to a new one. If the first number was smaller in absolute value, he subtracted the smaller number from the larger number. If the first number was larger in absolute value, then he added the two numbers. He used this rule for 8 items (all except for the L-S and S-L types). His performance of the sign operation was inconsistent and undetermined.
- Subtest 4 -- His rule changed again. This time he changed the operation sign "-" to "+" and applied the right addition rule to items having explicit signs in the second number.

The tests after 2 weeks of classroom instruction was administered. (Test 5):

Classroom instruction started with an explanation of the concept of the number line. After students mastered the addition skills based on the number line method, the teachers switched their instruction to the use of verbal rules (Birenbaum & Tatsuoka, 1980; Chaiklin, 1982). Therefore, subtraction problems are taught by the use of the verbal rules.

- Subtest 1 -- He learned to use the right rule.
- Subtest 2 -- He applied the right rule consistently for taking the absolute value to the items having explicit signs in the second numbers. He used the correct sign for all items.
- Subtest 3 -- In the second subtest, he used the right rule consistently when taking the signs in the answers for all items. Taking the absolute values was done correctly for a subset of the items (items with parentheses and L-S, S-L types).
- Subtest 4 -- He used the right rule for all items.

The test after 2 weeks of classroom instruction was completed (Test 6):

- Subtest 1 -- He used the right rule for all items except #12.
- Subtest 2 -- He applied the right rule successfully to all the items.
- Subtest 3 -- The result was the same as in Subtest 2.
- Subtest 4 -- He used the right rule for a subset of items, except items 50 and 56.

Student A's performances are plotted into Figure 6.

Student B

Pretest (Test 3): She studied postman stories written on the PLATO system for about an hour in January 1980. She had not been exposed to any kind of instruction related to signed numbers. At the beginning of the 1980-81 fiscal year she took a 64-item signed-number test along with 40 other eighth graders before the revised version of postman stories was given to her class. Her diagnosed rules are as follows:

- Subtest 1 -- Her rule for taking an absolute value in the answer was undetermined. The signs of her answers for the items which don't have parentheses are yielded by the right rule. The random nature of her answers suggest she was not sure what she should do with the parentheses.

- Subtest 2 -- Her rule for taking an absolute value in the answer was undetermined. She used the right rule for items whose signs of the second number were not explicitly written (hidden sign).
- Subtest 3 -- Her rule for taking an absolute value in the answer was again undetermined. For the sign part, she changed her rule and completed the subtraction operation by disregarding the step of changing the sign of the second number. Thus, she changed the sign of the operation "-" to "+" and applied the right rule for addition problems to the newly converted addition problems. However, her rule was not applied consistently for the items having hidden signs in the second number.
- Subtest 4 -- Her performance was identical to her performance on Subtest 3.

The test given after 1 hour PLATO lesson was studied (Test 4):

- Subtest 1 -- She applied the right rule to the items with parentheses and L-S, S-L types.
- Subtest 2 -- She used the right rule for taking an absolute value and obtained the right answers for the items with the parentheses and L-S, S-L types. But the rule for taking signs to the answers was not consistent so the rule was undetermined.
- Subtest 3 -- She answered all 10 items with the right rule.
- Subtest 4 -- She used the right rule for nine items except for -L - (-S) type. Her error is due to mistyping a sign in the answer.

The test after the classroom instruction (2 weeks) was given (Test 5):

- Subtest 1 -- Her rule regressed. She did not change the sign of the second number at all. Instead, she changed the operation sign "-" to "+" and consistently applied the right rule for items having explicit signs in the second number. second numbers.
- Subtest 2 -- She used the same rule described above. But the rule of taking an absolute value in the answer was not consistent but her sign operation was consistent for the items whose signs were explicit in the second number.
- Subtest 4 -- Her performance was identical to the performances on Subtest 3.

The test given after multiplication and division of signed numbers (Test 6):
She applied the right rule repeatedly over the four subtests and answered correctly for all 40 items in the test. This student's performance is shown in Figure 7.

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